



Neutrosophic Q -Ideals of Neutrosophic Q -Algebra*

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Abstract: the study of neutrosophic sets in different fields has been growing in the last decade; it allows the modelling and studying of some physical, biological and behavioral phenomena. In this paper, we establish the notion of neutrosophic Q -ideal of neutrosophic Q -algebra, which is an extension of some results presented in fuzzy and intuitionistic fuzzy sets. Finally, we formulate several conclusions.

Keywords: q -algebra; q -ideal; q -sub-algebra; fuzzy set; intuitionistic fuzzy set; neutrosophic set

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Q-Ideales neutrosóficos de Q-Álgebra neutrosófica

Resumen: el estudio de los conjuntos neutrosóficos en diferentes campos ha ido creciendo en la última década, esto permite modelar y estudiar algunos fenómenos físicos, biológicos y de comportamiento. En este artículo, establecemos la noción de Q-ideal neutrosófico del Q-álgebra neutrosófica, que es una extensión de algunos resultados presentados en conjuntos borrosos e intuicionistas. Finalmente, formulamos varias conclusiones.

Palabras clave: q-álgebra; q-ideal; q-sub-álgebra; conjunto borroso; conjunto borroso intuicionista; conjunto neutrosófico

Q-Ideals Neutrosóficos da Álgebra Q Neutrosófica

Resumo: o estudo de conjuntos neutrosóficos em diferentes áreas vem crescendo na última década; permite modelar e estudar alguns fenômenos físicos, biológicos e comportamentais. Neste artigo, estabelecemos a noção de Q-ideal neutrosóficos da álgebra-Q neutrosófica, que é uma extensão de alguns resultados apresentados em conjuntos fuzzy e intuicionistas. Por fim, formulamos várias conclusões.

Palavras-chave: q-álgebra; q-ideal; q-sub-álgebra; conjunto difuso; conjunto fuzzy intuicionista; conjunto neutrosófico

Introduction

In 1978, [1] grounded the notion of BCK-algebra. Then, [2] presented d -algebra as a generalization of BCK-algebra. In 1999, [3] studied the notion of d -ideal in d -algebra. The idea of fuzzy set (FS) theory was described by [4] in 1965. Later, [5] generalized the concept of FS by presenting the idea of intuitionistic fuzzy set (IFS) theory. [6] studied fuzzy d -ideals of d -algebras in 2000. The notion of intuitionistic fuzzy d -algebra was established by [7]. In 2017, the idea of intuitionistic fuzzy d -ideal of d -algebra was introduced by [8]. [9] also grounded the idea of intuitionistic fuzzy d -filter of d -algebra. [10] introduced and studied the notion of Q -algebra in 2001. [11] presented some new types of ideals in Q -algebra. [12] grounded the idea of fuzzy Q -ideals in Q -algebras. [13] also introduced the notion of intuitionistic fuzzy Q -ideals of Q -algebra. In 2005, [14] presented the neutrosophic set as an extension of IFS. Subsequently, [15] introduced the concept of neutrosophic BCI/BCK -algebras. In 2016, [16] presented the single valued neutrosophic ideals of BCK -algebras. Recently, Das et al. [17] defined and studied the Pentapartitioned Neutrosophic Q -ideals of Q -algebra. In this article, we look at the notion of neutrosophic Q -algebra and neutrosophic Q -ideal.

The rest of the paper is designed as follows:

In Section 2, we recall Q -algebra, Q -ideal, fuzzy Q -algebra, fuzzy Q -ideal, intuitionistic fuzzy Q -algebra, intuitionistic fuzzy Q -ideal. In section 3, we present the concept of neutrosophic Q -algebra of neutrosophic Q -ideal by extending the concept of intuitionistic fuzzy Q -algebra. Further, we formulate some propositions, remarks, and theorems on neutrosophic Q -ideals of Q -algebra. In section 4, we present the concluding remarks of the paper.

Preliminaries and definitions

Definition 2.1. [1] Suppose that W is a fixed set. Let 0 be a constant in W and $*$ be a binary operation defined on W . Then $(W, *, 0)$ is called a BCK-algebra if the following holds:

- (i) $((x * y) * (x * z)) * (z * y) = 0$,
- (ii) $(x * (x * y)) * y = 0$,
- (iii) $x * x = 0$,
- (iv) $x * y = y * x = 0 \Rightarrow x = y$,
- (v) $0 * x = 0$, for all $x, y, z \in X$.

Definition 2.2. [1] Let W be a BCK-algebra with binary operator $*$ and a constant 0 . Then $I \subseteq W$ is called a BCK-ideal of W if the following holds:

- (i) $0 \in I$;
- (ii) $h * d \in I$ and $d \in I \Rightarrow h \in I, \forall h, d \in W$.

Definition 2.3. [10] Suppose that W is a fixed set. Let 0 be a constant in W and $*$ be a binary operation defined on W . Then $(W, *, 0)$ is called a Q -algebra if the followings hold:

- (i) $h * h = 0, \forall h \in W$
- (ii) $0 * h = h = h * 0, \forall h \in W$
- (iii) $(h * d) * e = (h * e) * d, \forall h, d, e \in W$.

Sometime, one can refer to $h \leq d$ iff $h * d = 0$.

Definition 2.4. [10] Let $(W, *, 0)$ be a Q -algebra. Then, $(W, *, 0)$ is said to be commutative Q -algebra if $c * (c * d) = d * (d * c), \forall c, d \in W$, and $d * (d * c)$ is denoted by $(c \wedge d)$.

Definition 2.5. [10] A Q -algebra W is bounded if there exist $g \in W$ such that $h \leq g$ for all $h \in W$, i.e. $h * g = 0, \forall h \in W$.

Definition 2.6. [10] Let W be a Q -algebra with binary operator $*$ and $H (\neq 0_N) \subseteq W$. Then, H is called a Q -sub-algebra of W , if $h, d \in H$ implies $h * d \in H$.

Definition 2.7. [11] Let W be a Q -algebra with binary operator $*$ and a constant 0 . Then, $I \subseteq W$ is called a Q -ideal of W if the following holds:

- (i) $0 \in I$;
- (ii) $(h * d) * e \in I$ and $d \in I \Rightarrow h * e \in I$, for all $h, d, e \in W$.

Proposition 2.1. [11] Let $(W, *, 0)$ be called a Q -algebra. Let I be a Q -ideal of W . Then, I be a BCK-ideal of W .

Definition 2.8. [18] A fuzzy set $Y = \{(c, T_Y(c)) : c \in W\}$ over a BCK-algebra W is called the fuzzy BCK-ideal if the following two conditions holds:

- (i) $T_Y(0) \geq T_Y(c)$, for all $c \in W$;
- (ii) $T_Y(c) \geq \min\{T_Y((c * d), T_Y(d))\}$ for all $c, d \in W$.

Definition 2.9. [12] A fuzzy set $Y = \{(c, T_Y(c)) : c \in W\}$ over a Q -algebra W is called the fuzzy Q -ideal if the following holds:

- (i) $T_Y(0) \geq T_Y(c)$, for all $c \in W$;
- (ii) $T_Y(c * d) \geq \min\{T_Y((c * h) * d), T_Y(h)\}$ for all $c, h, d \in W$.

Lemma 2.1. [12] Let $(W, *, 0)$ be a Q -algebra. If $Y = \{(c, T_Y(c)) : c \in W\}$ is a fuzzy Q -ideal of W , then it is also a fuzzy BCK-ideal of W .

Definition 2.10. [13] An intuitionistic fuzzy set $Y = \{(c, T_Y(c), F_Y(c)) : c \in W\}$ over a Q -algebra W is called the intuitionistic fuzzy Q -ideal if it satisfies the following inequalities:

- (i) $T_Y(0) \geq T_Y(c)$, $F_Y(0) \leq F_Y(c)$, for all $c \in W$;
- (ii) $T_Y(c * d) \geq \min\{T_Y((c * h) * d), T_Y(h)\}$;
- (iii) $F_Y(c * d) \leq \max\{F_Y((c * h) * d), F_Y(h)\}$.

Lemma 2.2. [13] Let $Y = \{(c, T_Y(c), F_Y(c)) : c \in W\}$ be an intuitionistic fuzzy Q -ideal over a Q -algebra W . If $c * d \leq h$, for all $c, d, h \in W$, then $T_Y(c) \geq \min\{T_Y(d), T_Y(h)\}$ and $F_Y(c) \geq \max\{F_Y(d), F_Y(h)\}$, for all $c \in W$.

Lemma 2.3. [13] Let $Y = \{(c, T_Y(c), F_Y(c)) : c \in W\}$ be an intuitionistic fuzzy Q -ideal over a Q -algebra W . If $c \leq d$, for all $c, d \in W$, then $T_Y(c) \geq T_Y(d)$ and $F_Y(c) \leq F_Y(d)$.

Lemma 2.4. [13] If $Y = \{(c, T_Y(c), F_Y(c)) : c \in W\}$ be an intuitionistic fuzzy Q -ideal over a Q -algebra W , then the sets $\alpha\text{-}T_Y = \{c : c \in W, T_Y(c) \geq \alpha\}$ and $\alpha\text{-}F_Y = \{c : c \in W, F_Y(c) \leq \alpha\}$ are Q -ideal of Q -algebra W .

Definition 2.11. [14] A neutrosophic set over a universal set W is defined as follows:

$H = \{(y, T_H(y), I_H(y), F_H(y)) : y \in W\}$, where $T_H(y)$, $I_H(y)$ and $F_H(y)$ ($\in [0, 1]$) are the truth, indeterminacy and false membership value of y and $0 \leq T_H(y) + I_H(y) + F_H(y) \leq 3$.

Definition 2.12. [14] The neutrosophic whole set (1_N) and neutrosophic null set (0_N) over a universal set W are defined by:

- (i) $1_N = \{(y, 1, 0, 0) : y \in W\}$.
- (ii) $0_N = \{(y, 0, 0, 1) : y \in W\}$.

Definition 2.13. [14] [19] Assume that $H = \{(y, T_H(y), I_H(y), F_H(y)) : y \in W\}$ and $K = \{(y, T_K(y), I_K(y), F_K(y)) : y \in W\}$ are any two neutrosophic sets over X . Then

- (i) $H^c = \{(y, 1 - T_H(y), 1 - I_H(y), 1 - F_H(y)) : y \in W\}$;
- (ii) $H \subseteq K \Leftrightarrow T_H(y) \leq T_K(y), I_H(y) \geq I_K(y), F_H(y) \geq F_K(y)$, for each $y \in W$.
- (iii) $H \cup K = \{(y, T_H(y) \vee T_K(y), I_H(y) \wedge I_K(y), F_H(y) \wedge F_K(y)) : y \in W\}$;
- (iv) $H \cap K = \{(y, T_H(y) \wedge T_K(y), I_H(y) \vee I_K(y), F_H(y) \vee F_K(y)) : y \in W\}$;

Neutrosophic Q -Ideals

In this section, we establish the concept of neutrosophic Q -ideals of neutrosophic Q -algebra by expanding the notion of intuitionistic fuzzy Q -ideals. Then, we formulate some results on neutrosophic Q -ideal and neutrosophic Q -algebra.

Definition 3.1. Let $Y = \{< c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) : c \in W\}$ be a NS over a Q -algebra W . Then, Y is called a neutrosophic Q -algebra if the following conditions hold:

- (i) $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$;
- (ii) $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\}$;
- (iii) $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, where $c, d \in W$.

By the structure $(N< W, Y, *, 0)$ we denote the neutrosophic Q-algebra Y over W.

Theorem 3.1. Let $\{Y_i: i \in \Delta\}$ be the family of neutrosophic Q-algebra of W. Then, $\bigcap_{i \in \Delta} Y_i$ is a neutrosophic Q-algebra of W.

Proof. Let $\{Y_i: i \in \Delta\}$ be a collection of neutrosophic Q-algebras of W. It is known that $\bigcap_{i \in \Delta} Y_i = \{< c, \wedge \hat{A}_{Y_i}(c), \vee \hat{E}_{Y_i}(c), \vee \hat{U}_{Y_i}(c) > : c \in W\}$.

Now,

$$\begin{aligned} \wedge \hat{A}_Y(c * d) &= \wedge \{\hat{A}_{Y_i}((c * d): i \in \Delta) \\ &\geq \wedge \{\min\{\hat{A}_{Y_i}((c * h) * d), \hat{A}_{Y_i}(h)\}\} \\ &= \min\{\wedge \hat{A}_{Y_i}((c * h) * d), \wedge \hat{A}_{Y_i}(h)\} \\ \Rightarrow \wedge \hat{A}_Y(c * d) &\geq \min\{\wedge \hat{A}_{Y_i}((c * h) * d), \wedge \hat{A}_{Y_i}(h)\}. \end{aligned}$$

Now,

$$\begin{aligned} \vee \hat{E}_Y(c * d) &= \vee \{\hat{E}_{Y_i}(c * d): i \in \Delta\} \\ &\geq \vee \{\min\{\hat{E}_{Y_i}((c * h) * d), \hat{E}_{Y_i}(h)\}\} \\ &= \min\{\vee \hat{E}_{Y_i}((c * h) * d), \vee \hat{E}_{Y_i}(h)\} \\ \Rightarrow \vee \hat{E}_Y(c * d) &\geq \min\{\vee \hat{E}_{Y_i}((c * h) * d), \vee \hat{E}_{Y_i}(h)\}. \end{aligned}$$

Now,

$$\begin{aligned} \vee \hat{U}_Y(c * d) &= \vee \{\hat{U}_{Y_i}(c * d): i \in \Delta\} \\ &\geq \vee \{\min\{\hat{U}_{Y_i}((c * h) * d), \hat{U}_{Y_i}(h)\}\} \\ &= \min\{\vee \hat{U}_{Y_i}((c * h) * d), \vee \hat{U}_{Y_i}(h)\} \\ \Rightarrow \vee(c * d) &\geq \min\{\vee((c * h) * d), \vee(h)\}. \end{aligned}$$

Hence, is a neutrosophic Q-algebra of W.

Definition 3.2. Let $Y = \{< c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) > : c \in W\}$ be a NS over a Q-algebra W. Then, Y is called a neutrosophic Q-sub-algebra if the following conditions hold:

- (i) $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\};$
- (ii) $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\};$

(iii) $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\};$ where $c, d \in W$.

Theorem 3.2. Suppose that $Y = \{< c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) > : c \in W\}$ be a neutrosophic Q-sub-algebra of a Q-algebra W. Then,

- (i) $\hat{A}_Y(0) \geq \hat{A}_Y(c),$ for all $c \in W;$
- (ii) $\hat{E}_Y(0) \leq \hat{E}_Y(c),$ for all $c \in W;$
- (iii) $\hat{U}_Y(0) \leq \hat{U}_Y(c),$ for all $c \in W.$

Proof. Suppose that $Y = \{< c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) > : c \in W\}$ be a neutrosophic Q-sub-algebra of a Q-algebra W. Therefore, $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}, \hat{E}_Y(c * d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}, \hat{U}_Y(c * d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\},$ for all $c, d \in W$.

Now we have,

$$\begin{aligned} \hat{A}_Y(0) &= \hat{A}_Y(c * c) \\ &\geq \min\{\hat{A}_Y(c), \hat{A}_Y(c)\} \\ &= \hat{A}_Y(c) \\ \Rightarrow \hat{A}_Y(0) &\geq \hat{A}_Y(c), \text{ for all } c \in W. \\ \hat{E}_Y(0) &= \hat{E}_Y(c * c) \\ &\leq \max\{\hat{E}_Y(c), \hat{E}_Y(c)\} \\ &= \hat{E}_Y(c) \\ \Rightarrow \hat{E}_Y(0) &\leq \hat{E}_Y(c), \text{ for all } c \in W. \\ \hat{U}_Y(0) &= \hat{U}_Y(c * c) \\ &\leq \max\{\hat{U}_Y(c), \hat{U}_Y(c)\} \\ &= \hat{U}_Y(c) \\ \Rightarrow \hat{U}_Y(0) &\leq \hat{U}_Y(c), \text{ for all } c \in W. \end{aligned}$$

Definition 3.3. A neutrosophic set $Y = \{< c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) > : c \in W\}$ over a Q-algebra W is called the neutrosophic Q-ideal if and only if the following inequalities hold:

- (i) $\hat{A}_Y(0) \geq \hat{A}_Y(c) \ \& \ \hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\},$ for all $c, d, h \in W;$

(ii) $\hat{E}_Y(0) \leq \hat{E}_Y(c) \ \& \ \hat{E}_Y(c * d) \leq \max \{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\}$, for all $c, d, h \in W$;

(iii) $\hat{U}_Y(0) \leq \hat{U}_Y(c) \ \& \ \hat{U}_Y(c * d) \leq \max \{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, for all $c, d, h \in W$.

Remark 3.1. Every neutrosophic Q -ideal of a Q -algebra W is a neutrosophic Q -sub-algebra.

Theorem 3.3. If $\{D_i; i \in \Delta\}$ be the collection of neutrosophic Q -ideals of Q -algebra W , then $\bigcap_{i \in \Delta} D_i$ is also a neutrosophic Q -ideal of Q -algebra W .

Proof. Suppose that $\{D_i; i \in \Delta\}$ be a family of neutrosophic Q -ideals of Q -algebra W . Therefore,

(i) $\hat{A}_{D_i}(0) \geq \hat{A}_{D_i}(c) \ \& \ (c * d) \geq \min \{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$, $\hat{A}_{D_i}$ for all $c, d, h \in W$ and $i \in \Delta$.

(ii) $\hat{E}_{D_i}(0) \leq \hat{E}_{D_i}(c) \ \& \ \hat{E}_{D_i}(c * d) \leq \max \{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

(iii) $\hat{U}_{D_i}(0) \leq \hat{U}_{D_i}(c) \ \& \ \hat{U}_{D_i}(c * d) \leq \max \{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

It is known that $\bigcap_{i \in \Delta} D_i = \{c, \wedge \hat{A}_{D_i}(c), \vee \hat{E}_{D_i}(c), \vee \hat{U}_{D_i}(c) : c \in W\}$.

Now, we have

$$\hat{A}_{D_i}(0) \geq \hat{A}_{D_i}(c), \text{ for all } c \in W \text{ and } i \in \Delta$$

$$\Rightarrow \wedge \hat{A}_{D_i}(0) \geq \wedge \hat{A}_{D_i}(c).$$

$$\hat{E}_{D_i}(0) \leq \hat{E}_{D_i}(c), \text{ for all } c \in W \text{ and } i \in \Delta$$

$$\Rightarrow \vee \hat{E}_{D_i}(0) \leq \vee \hat{E}_{D_i}(c).$$

$$\hat{U}_{D_i}(0) \leq \hat{U}_{D_i}(c), \text{ for all } c \in W \text{ and } i \in \Delta$$

$$\Rightarrow \vee \hat{U}_{D_i}(0) \leq \vee \hat{U}_{D_i}(c).$$

Further, we have

$$\hat{A}_{D_i}(c * d) \geq \min \{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}, \text{ for all } c, d, h \in W \text{ and } i \in \Delta.$$

$$\Rightarrow \wedge \hat{A}_{D_i}(c * d) \geq \wedge \min \{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$$

$$= \min \{\wedge \hat{A}_{D_i}((c * h) * d), \wedge \hat{A}_{D_i}(h)\}$$

$$\Rightarrow \wedge \hat{A}_{D_i}(c * d) \geq \min \{\wedge \hat{A}_{D_i}((c * h) * d), \wedge \hat{A}_{D_i}(h)\}.$$

$$\hat{E}_{D_i}(c * d) \leq \max \{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}, \text{ for all } c, d, h \in W \text{ and } i \in \Delta.$$

$$\Rightarrow \vee \hat{E}_{D_i}(c * d) \leq \vee \max \{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$$

$$= \max \{\vee \hat{E}_{D_i}((c * h) * d), \vee \hat{E}_{D_i}(h)\}$$

$$\Rightarrow \vee \hat{E}_{D_i}(c * d) \leq \max \{\hat{E}_{D_i} \vee ((c * h) * d), \vee \hat{E}_{D_i}(h)\}.$$

$$\hat{U}_{D_i}(c * d) \leq \max \{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}, \text{ for all } c, d, h \in W \text{ and } i \in \Delta.$$

$$\Rightarrow \vee \hat{U}_{D_i}(c * d) \leq \vee \max \{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$$

$$= \max \{\vee \hat{U}_{D_i}((c * h) * d), \vee \hat{U}_{D_i}(h)\}$$

$$\Rightarrow \vee \hat{U}_{D_i}(c * d) \leq \max \{\vee \hat{U}_{D_i}((c * h) * d), \vee \hat{U}_{D_i}(h)\}.$$

Hence, $\bigcap_{i \in \Delta} D_i$ is a neutrosophic Q -ideal of Q -algebra W .

Proposition 3.1. Let $Y = \{c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) : c \in W\}$ be a neutrosophic Q -ideal of a Q -algebra W . Then, Y is a neutrosophic BCK-ideal of the BCK-algebra W .

Theorem 3.4. Suppose that $Y = \{c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) : c \in W\}$ is a neutrosophic Q -ideal over a Q -algebra W . If $c, h, d \in W$ such that $c * h \leq d$, then $\hat{A}_Y(c) \geq \min \{\hat{A}_Y(h), \hat{A}_Y(d)\}$, $\hat{E}_Y(c) \leq \max \{\hat{E}_Y(h), \hat{E}_Y(d)\}$, and $\hat{U}_Y(c) \leq \max \{\hat{U}_Y(h), \hat{U}_Y(d)\}$.

Proof. Let $Y = \{c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) : c \in W\}$ be a neutrosophic Q -ideal over a Q -algebra W . Suppose $c, h, d \in W$ such that $c * h \leq d$. Therefore, $(c * h) * d = 0$.

Now, we have

$$\hat{A}_Y(c) = \hat{A}_Y(c * 0) \geq \min \{\hat{A}_Y((c * h) * 0), \hat{A}_Y(h)\}$$

$$= \min \{\hat{A}_Y((c * h)), \hat{A}_Y(h)\}$$

$$\geq \min \{\min \{\hat{A}_Y((c * d) * h), \hat{A}_Y(d)\}, \hat{A}_Y(h)\}$$

$$= \min \{\hat{A}_Y((c * h) * d), \hat{A}_Y(d),$$

$$\hat{A}_Y(h)\}$$

$$= \min \{\hat{A}_Y(0), \hat{A}_Y(d), \hat{A}_Y(h)\}$$

$$= \min \{\hat{A}_Y(h), \hat{A}_Y(d)\}$$

$$\Rightarrow \hat{A}_Y(c) \geq \min \{\hat{A}_Y(h), \hat{A}_Y(d)\}.$$

Now, we have

$$\hat{E}_Y(c) = \hat{E}_Y(c * 0) \leq \max \{\hat{E}_Y((c * h) * 0), \hat{E}_Y(h)\}$$

$$\begin{aligned}
 &= \max \{ \hat{E}_Y((c * h)), \hat{E}_Y(h) \} \\
 &\leq \max \{ \max \{ \hat{E}_Y((c * d) * h), \hat{E}_Y(d) \}, \hat{E}_Y(h) \} \\
 &= \max \{ \hat{E}_Y((c * h) * d), \hat{E}_Y(d), \\
 &\hat{E}_Y(h) \} \\
 &= \max \{ \hat{E}_Y(0), \hat{E}_Y(d), \hat{E}_Y(h) \} \\
 &= \max \{ \hat{E}_Y(h), \hat{E}_Y(d) \} \\
 \Rightarrow \hat{E}_Y(c) &\leq \max \{ \hat{E}_Y(h), \hat{E}_Y(d) \}.
 \end{aligned}$$

Further, we have

$$\begin{aligned}
 \hat{U}_Y(c) &= \hat{U}_Y(c * 0) \leq \max \{ \hat{U}_Y((c * h) * 0), \hat{U}_Y(h) \} \\
 &= \max \{ \hat{U}_Y((c * h)), \hat{U}_Y(h) \} \\
 &\leq \max \{ \max \{ \hat{U}_Y((c * d) * h), \hat{U}_Y(d) \}, \\
 &\hat{U}_Y(h) \} \\
 &= \max \{ \hat{U}_Y((c * h) * d), \hat{U}_Y(d), \\
 &\hat{U}_Y(h) \} \\
 &= \max \{ \hat{U}_Y(0), \hat{U}_Y(d), \hat{U}_Y(h) \} \\
 &= \max \{ \hat{U}_Y(h), \hat{U}_Y(d) \} \\
 \Rightarrow \hat{U}_Y(c) &\leq \max \{ \hat{U}_Y(h), \hat{U}_Y(d) \}.
 \end{aligned}$$

Theorem 3.5. Suppose that $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ is a neutrosophic Q -ideal over a Q -algebra W . If $c, h \in W$ such that $c \leq h$, then $\hat{A}_Y(c) \geq \hat{A}_Y(h)$, $\hat{E}_Y(c) \leq \hat{E}_Y(h)$ and $\hat{U}_Y(c) \leq \hat{U}_Y(h)$.

Proof. Let $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ be a neutrosophic Q -ideal over a Q -algebra W . Suppose c, h be two elements of W such that $c \leq h$. Therefore, $c * h = 0$.

Now, we have

$$\begin{aligned}
 \hat{A}_Y(c) &= \hat{A}_Y(c * 0) \geq \min \{ \hat{A}_Y((c * h) * 0), \hat{A}_Y(h) \} \\
 &= \min \{ \hat{A}_Y((c * h)), \hat{A}_Y(h) \} \\
 &= \min \{ \hat{A}_Y(0), \hat{A}_Y(h) \} \\
 &= \hat{A}_Y(h) \\
 \Rightarrow \hat{A}_Y(c) &\geq \hat{A}_Y(h).
 \end{aligned}$$

Now, we have

$$\begin{aligned}
 \hat{E}_Y(c) &= \hat{E}_Y(c * 0) \leq \max \{ \hat{E}_Y((c * h) * 0), \hat{E}_Y(h) \} \\
 &= \max \{ \hat{E}_Y((c * h)), \hat{E}_Y(h) \} \\
 &= \max \{ \hat{E}_Y(0), \hat{E}_Y(h) \} \\
 &= \hat{E}_Y(h) \\
 \Rightarrow \hat{E}_Y(c) &\leq \hat{E}_Y(h).
 \end{aligned}$$

Further, we have

$$\begin{aligned}
 \hat{U}_Y(c) &= \hat{U}_Y(c * 0) \leq \max \{ \hat{U}_Y((c * h) * 0), \hat{U}_Y(h) \} \\
 &= \max \{ \hat{U}_Y((c * h)), \hat{U}_Y(h) \} \\
 &= \max \{ \hat{U}_Y(0), \hat{U}_Y(h) \} \\
 &= \hat{U}_Y(h) \\
 \Rightarrow \hat{U}_Y(c) &\leq \hat{U}_Y(h).
 \end{aligned}$$

Theorem 3.6. If $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ is a neutrosophic Q -sub-algebra over a Q -algebra W , then the sets $\alpha\text{-}\hat{A}_Y = \{ c : c \in W, \hat{A}_Y(c) \geq \alpha \}$ and $\alpha\text{-}\hat{E}_Y = \{ c : c \in W, \hat{E}_Y(c) \leq \alpha \}$, and $\alpha\text{-}\hat{U}_Y = \{ c : c \in W, \hat{U}_Y(c) \leq \alpha \}$ are the Q -sub-algebra of W .

Proof. Suppose that $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ is a neutrosophic Q -sub-algebra over a Q -algebra W . Therefore,

- (i) $\hat{A}_Y(c * d) \geq \min \{ \hat{A}_Y(c), \hat{A}_Y(d) \}$;
- (ii) $\hat{E}_Y(c * d) \leq \max \{ \hat{E}_Y(c), \hat{E}_Y(d) \}$;
- (iii) $\hat{U}_Y(c * d) \leq \max \{ \hat{U}_Y(c), \hat{U}_Y(d) \}$; where $c, d \in W$.

Let $c, d \in \alpha\text{-}\hat{A}_Y$. This implies, $\hat{A}_Y(c) \geq \alpha$, $\hat{A}_Y(d) \geq \alpha$.

Therefore, $\hat{A}_Y(c * d) \geq \min \{ \hat{A}_Y(c), \hat{A}_Y(d) \} \geq \min \{ \alpha, \alpha \} \geq \alpha$.

Hence, $\alpha\text{-}\hat{A}_Y = \{ c : c \in W, \hat{A}_Y(c) \geq \alpha \}$ is a Q -sub-algebra of W .

Let $c, d \in \alpha\text{-}\hat{E}_Y$. This implies, $\hat{E}_Y(c) \leq \alpha$, $\hat{E}_Y(d) \leq \alpha$.

Therefore, $\hat{E}_Y(c * d) \leq \max \{ \hat{E}_Y(c), \hat{E}_Y(d) \} \leq \max \{ \alpha, \alpha \} \leq \alpha$.

Hence, $\alpha\text{-}\hat{E}_Y = \{ c : c \in W, \hat{E}_Y(c) \leq \alpha \}$ is a Q -sub-algebra of W .

Let $c, d \in \alpha - \hat{U}_Y$. This implies, $\hat{U}_Y(c) \leq \alpha$, $\hat{U}_Y(d) \leq \alpha$.

Therefore, $\hat{U}_Y(c * d) \leq \max \{ \hat{U}_Y(c), \hat{U}_Y(d) \} \leq \max \{ \alpha, \alpha \} \leq \alpha$.

Hence, $\alpha - \hat{U}_Y = \{c \in W, \hat{U}_Y(c) \leq \alpha\}$ is a Q-sub-algebra of W.

Theorem 3.7. If $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ is a neutrosophic Q-ideal of W, then the sets $W(\hat{A}) = \{c \in W: \hat{A}_Y(c) = \hat{A}_Y(0)\}$, $W(\hat{E}) = \{c \in W: \hat{E}_Y(c) = \hat{E}_Y(0)\}$, and $W(\hat{U}) = \{c \in W: \hat{U}_Y(c) = \hat{U}_Y(0)\}$ are Q-ideals of W.

Proof. Suppose that $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ is a neutrosophic Q-ideal of W. Therefore,

(i) $\hat{A}_Y(0) \geq \hat{A}_Y(c) \ \& \ \hat{A}_Y(c * d) \geq \min \{ \hat{A}_Y((c * h) * d), \hat{A}_Y(h) \}$, for all $c, d, h \in W$;

(ii) $\hat{E}_Y(0) \leq \hat{E}_Y(c) \ \& \ \hat{E}_Y(c * d) \leq \max \{ \hat{E}_Y((c * h) * d), \hat{E}_Y(h) \}$, for all $c, d, h \in W$;

(iii) $\hat{U}_Y(0) \leq \hat{U}_Y(c) \ \& \ \hat{U}_Y(c * d) \leq \max \{ \hat{U}_Y((c * h) * d), \hat{U}_Y(h) \}$, for all $c, d, h \in W$.

Since, $\hat{A}_Y(0) = \hat{A}_Y(0)$, so $0 \in W(\hat{A})$.

Since, $\hat{E}_Y(0) = \hat{E}_Y(0)$, so $0 \in W(\hat{E})$.

Since, $\hat{U}_Y(0) = \hat{U}_Y(0)$, so $0 \in W(\hat{U})$.

Let $(h * d) * e \in W(\hat{A})$ and $d \in W(\hat{A})$. Therefore, $\hat{A}_Y((h * d) * e) = \hat{A}_Y(0)$ and $\hat{A}_Y(d) = \hat{A}_Y(0)$.

It is clear that $\hat{A}_Y(0) \geq \hat{A}_Y(h * e)$ (1)

Now, we have

$\hat{A}_Y(h * e) \geq \min \{ \hat{A}_Y((h * d) * e), \hat{A}_Y(d) \} = \min \{ \hat{A}_Y(0), \hat{A}_Y(0) \} = \hat{A}_Y(0)$

$\Rightarrow \hat{A}_Y(h * e) \geq \hat{A}_Y(0)$ (2)

From (1) and (2), we get

$\hat{A}_Y(h * e) = \hat{A}_Y(0)$.

This implies $h * e \in W(\hat{A})$. Therefore, the set $W(\hat{A}) = \{c \in W: \hat{A}_Y(c) = \hat{A}_Y(0)\}$ is a Q-ideal of W.

Similarly, it can be shown that, the sets $W(\hat{E}) = \{c \in W: \hat{E}_Y(c) = \hat{E}_Y(0)\}$, and $W(\hat{U}) = \{c \in W: \hat{U}_Y(c) = \hat{U}_Y(0)\}$ are Q-ideals of W.

Theorem 3.8. Let $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ be a neutrosophic Q-ideal of Q-algebra W. Then, the fuzzy sets $\{(c, \hat{A}_Y(c)): c \in W\}$, $\{(c, 1 - \hat{E}_Y(c)): c \in W\}$, $\{(c, 1 - \hat{U}_Y(c)): c \in W\}$ are fuzzy Q-ideals of W.

Proof. Suppose that $Y = \{ \langle c, \hat{A}_Y(c), \hat{E}_Y(c), \hat{U}_Y(c) \rangle : c \in W \}$ is a neutrosophic Q-ideal of a Q-algebra W. Therefore,

(i) $\hat{A}_Y(0) \geq \hat{A}_Y(c) \ \& \ \hat{A}_Y(c * d) \geq \min \{ \hat{A}_Y((c * h) * d), \hat{A}_Y(h) \}$, for all $c, d, h \in W$;

(ii) $\hat{E}_Y(0) \leq \hat{E}_Y(c) \ \& \ \hat{E}_Y(c * d) \leq \max \{ \hat{E}_Y((c * h) * d), \hat{E}_Y(h) \}$, for all $c, d, h \in W$;

(iii) $\hat{U}_Y(0) \leq \hat{U}_Y(c) \ \& \ \hat{U}_Y(c * d) \leq \max \{ \hat{U}_Y((c * h) * d), \hat{U}_Y(h) \}$, for all $c, d, h \in W$.

It is clear that $\hat{A}_Y(0) \geq \hat{A}_Y(c) \ \& \ \hat{A}_Y(c * d) \geq \min \{ \hat{A}_Y((c * h) * d), \hat{A}_Y(h) \}$, for all $c, d, h \in W$. Therefore, the fuzzy set $\{(c, \hat{A}_Y(c)): c \in W\}$ is a fuzzy Q-ideal of W.

Now, for all $c, d, h \in W$,

$\hat{E}_Y(c * d) \leq \max \{ \hat{E}_Y((c * h) * d), \hat{E}_Y(h) \} \Rightarrow 1 - \hat{E}_Y(c) \geq \min \{ 1 - \hat{E}_Y((c * h) * d), 1 - \hat{E}_Y(h) \}$

and $\hat{E}_Y(0) \leq \hat{E}_Y(c) \Rightarrow 1 - \hat{E}_Y(0) \geq 1 - \hat{E}_Y(c)$.

Therefore, the fuzzy set $\{(c, 1 - \hat{E}_Y(c)): c \in W\}$ is a fuzzy Q-ideal of W.

Further, for all $c, d, h \in W$,

$\hat{U}_Y(c * d) \leq \max \{ \hat{U}_Y((c * h) * d), \hat{U}_Y(h) \} \Rightarrow 1 - \hat{U}_Y(c) \geq \min \{ 1 - \hat{U}_Y((c * h) * d), 1 - \hat{U}_Y(h) \}$

and $\hat{U}_Y(0) \leq \hat{U}_Y(c) \Rightarrow 1 - \hat{U}_Y(0) \geq 1 - \hat{U}_Y(c)$.

Therefore, the fuzzy set $\{(c, 1 - \hat{U}_Y(c)): c \in W\}$ is a fuzzy Q-ideal of W.

Conclusion

In this study, we have established the notion of neutrosophic Q-ideals of neutrosophic Q-algebra.

By defining neutrosophic Q -ideals, we have formulated some results on neutrosophic Q -algebra from the point of view of neutrosophic set. This is just the beginning of the concept of neutrosophic Q -algebra. In the future, we expect that many new investigations based on the notions of neutrosophic Q -ideals will be done.

Disclosure and conflict of interest

The authors declare that they have no conflicts of interest.

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