

The study of models and the understanding of reality

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If we consider modeling as an abstraction from reality that help us to understand and work with it, then mathematical modeling is as ancient that an specific date on which such practice began can not be accurately stated. According to Schichl [1], the use of numbers to refer to bones, and in that way, perform a mathematical representation of something real, dates from years before christian age. Actually, every culture have developed some sort of mathematical knowledge to account for daily life problems, because mathematics is inside all human relationships with the natural world and the social construction that allows us to live in community. Some examples of this are the optimal use of available space, that requires at least an intuitive understanding of geometry, and can be as complex as the most sophisticated architecture; the distribution of resources such as food, land or materials; the distribution of time and the recording of history through the calendar; and the management of social interactions through the established rules of the market.

For natural scientist, the language of mathematics has been crucial to develop useful models for specific problems and, from the study of those models, the creation (or development) of theories.

Physicists, for instance, understand the word through models and theories, that are supported on mathematical relationships among the measurable quantities related with the studied

phenomena. This mathematical rules aims to be suitable to predict future behaviors of the described system, under certain conditions.

When new data is found that may compromise the validity of previous models and theories, it is required to build a working explanation inside the accepted models or theories that can account for such data. If such explanation can not be reached, some sort of change is made on the models such that the new model or theory is enriched with the currently available information, and become a more solid way to understand the natural reality. The new model can be explored to find new predictions, new insights from the reality it represents. Here it is noticeable that the world have not suffer any changes, but the model we use to understand it, has.

Many scientist believe that “a model is a simplified version of something real”. Under such vision, the model is just a representation of reality, not reality itself. Under the process of refinement that models are subject to, a better model, that delivers a more accurate representation of reality, is found.

The mathematical rules inside the model can be understood as a manifestation of the presence of mathematics inside reality, so when a more comprehensive mathematical representation of the world is found, a better understanding of the nature is achieved.

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Under this formulation, the mathematical model evolves in time with the presence of new data coming from new theoretical or experimental techniques. Then, the mathematical part of the model is not a static representation of reality that is replaced as it became obsolete, but as an epistemic scientific object, evolves under the evolution in the understanding of the reality.

The view of Bezrucho and Smirnov [2] about models is quite more comprehensive, they understand as models anything that can be used to represent in some way an object of study. A model is something whose set of properties intersects the set of properties of the object that is modeled. So when we get to an understanding of the model, its properties and the way they relate, we can hope to have gotten an understanding over the object of study. More than that, they believe that models have a representation in ideas inside the mind, which constitute a model itself. This last model lives in the human mind and is the responsible of the comprehension we really have. In such matter than our understanding of the world is limited by the mind models we have, because without an idea present on the mind, the reality seems invisible. In this line of thoughts, a scientific model aims to explain the dynamical relationships among the properties of the modeled object, and the understanding of such scientific model depends on the mind models (ideas, concepts, beliefs) the person has. Under this vision, physics (or natural science in general) do study the models to understand the reality, but can not study the reality itself, because models are the link between what is in the mind (which is, what can be really comprehended) and the reality. Then, the model seems to be the direct epistemic scientific object (ESO), when reality is the indirect one.

In their entry in the Stanford Encyclopedia of Philosophy, Zalta et al. [3] provide an extensive review of several model classifications, making it clear that these classes are not incompatible with each other. Therefore, a model may fall into many of those categories simultaneously. They claim that models serve a cognitive function

because a significant part of scientific research is performed on the models created, not on the reality itself. This can be exemplified not only by the study of the hydrogen atom through quantum mechanical modeling but also by any ESO whose dimensions are far beyond the range of our senses, as it happens with astronomical or cosmological problems, as well as subatomic, atomic, or nano-scaled topics. This is possible because (according to Bezruchko and Smirnov), models can exhibit features similar to or coincident with those of the object being modeled. However, Zalta et al. seem to believe that translating the acquired knowledge from the model to the target system provides direct information and understanding of it, while Bezruchko and Smirnov state that what we call ‘understanding’ is only a mind model of reality, i.e., a model itself. In this line of thought, a scientific model aims to explain the dynamic relationships among the properties of the modeled object, and the understanding of such a scientific model depends on the mind models (ideas, concepts, beliefs) the person has. Under this vision, physics (or natural science in general) studies the models to understand reality but cannot study reality itself because models are the link between what is in the mind (which is what can be truly comprehended) and reality. Therefore, the model seems to be the direct epistemic scientific object, while reality is the indirect one.

In the book edited by Peschard [4], Weisberg states that the purpose of modeling is included inside the model, so different purposes must require different kinds of models representing the same real system, in such a way that each model intersects the system in a different set of properties. Also in that collection, Ronald Giere’s contribution discusses the experimental practice as a “model interpreted” one. Under this view, an experiment, which could be simplistically thought of as a way to obtain data from reality, is built from an initial model that deals with how the experiment is performed: what kind of materials, equipment, measurements, and what each measure would mean; the sequence of procedures to

be performed, etc. Under this perception, “the recorded data” (the direct output of the experiment) only makes sense under the “data model”, which is, in the sense of Bezruchko and Smirnov, the collection of ideas that drives the conceptualization of the meanings of such data.

It seems that scientific practice is always based on some sort of modeling, and that models are more than just a way to understand reality; they may be thought of as an ESO. But how do mathematical models evolve over time?

For instance, the mathematical modeling of physical problems has undergone several changes over time. In the early ages of modern physics, Isaac Newton presented the rules of mechanics in terms of differential equations [5], and it was understood that the comprehension of nature comes from the solution of such equations. This vision was changed later, in 1890, when Henri Poincaré showed that even if a closed analytical solution to the differential equations was not found, the relevant information about the physical system could be extracted from the analysis of the equations set. This seminal work led to the creation of a new way of modeling the mechanical world, which received the name “dynamical systems” (DS). It did not take long before the study of the dynamical equations transcended the field of physics and became of interest to mathematicians. As a consequence, several tools to study and gather information from such mathematical models were developed. So, DS became a tool to create and study models of any kind of problem, from engineering to medicine, from social sciences to the physics of crowd movement.

Nature is intrinsically complex, and the mathematical modeling of complexity requires new approaches. From the models of dynamical systems that establish a set of differential equations ruling the system, we move to agent-based models, where the rules of evolution act on every agent. From the interaction among agents, some exciting new features emerge that were not expected for the whole system. These new ways of modeling require strong computational capabilities, and their development could not be true

without the computational technologies. Of course, it was required more than a machine that can compute, but a way to code the mathematical algorithms was imperative. The computer revolution started with the IBM 7090, which could be instructed (coded) by using punched cards [6]. Its evolution has been quite fast, and from the rigid rules required to code in cards, coding languages evolved up to high-level languages like Python, which can be used in a way similar to written math, allowing the implementation of flexible and non-sequential algorithms, able to deal not only with the mathematics involved in the model, but also to simulate procedures applied in a similar fashion to what is experimentally done. This led to a new way of looking at science: computational science. Under this paradigm, the model, which before was required to have a small set of equations with easy-to-interpret meanings and easy to compute, can be upgraded to a model made of several correlated equations (hard to handle for human beings). Also, a set of procedures can be applied to the system by selecting different sets of rules referring to different stages. This last feature of computational models makes them similar to experimental setups. In fact, a set of simulations to explore the emergent features of a model can be understood as thought experiments. It is in this sense that several authors understand computational science as a third way of making science (experimental, theoretical, computational).

In conclusion, models are the tool used by science to understand natural reality, but they are also used to interpret the data coming from the outside world to the mind. A direct comprehension of reality is not possible, but a model of it must be built inside the mind. So what is comprehended is just a model of reality itself. Once this is accepted, the mutability of scientific theories comes naturally.

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